Light hadron spectrum with Kogut-Susskind quarks*

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We made an extensive study of the light hadron spectrum using the Wilson gauge action and Kogut-Susskind quarks. Using both dynamical quarks and the quenched approximation, we determine hadron masses for five and four gauge couplings respectively, with at least five quark masses at each coupling. In the continuum limit, we find a significant difference between two flavors of dynamical quarks and the quenched approximation for a range of quark masses.

Calculating the light quark spectrum is a long term goal of QCD. Because of limitations of the numerical approach, one must work with quark masses heavier than in Nature and with a non-zero lattice spacing. Good control over the extrapolations in quark mass and lattice spacing requires that simulations be carried out over a wide range of quark mass and lattice spacing. We have been studying the spectrum with Kogut-Susskind or staggered quarks for quite some time [1]. This year, we extended our dynamical data set and developed a new extrapolation method for the quenched case.

In the quenched approximation, in the continuum limit at the physical value of m_π/m_ρ, we find $m_N/m_\rho = 1.254 \pm 0.018 \pm 0.028$ [2], where the first error is statistical and the second is systematic. With dynamical quarks, the double extrapolation in quark mass and lattice spacing is difficult. However, we can confidently extrapolate to the continuum limit at intermediate values of the quark mass where we only need to interpolate in quark mass or extrapolate slightly from our lightest quark mass. For dynamical quarks at such intermediate quark masses, m_N/m_ρ is significantly larger than it is in the quenched approximation. We believe this is the first convincing evidence of the effects of dynamical quarks on the light quark spectrum. Before having confidence in our extrapolation to the chiral limit, we would need to have results for lighter quarks at our two weakest couplings. At Edinburgh [3], we presented our continuum extrapolation for the physical value of m_π/m_ρ and found it consistent with the experimental value. However, at that time we only had four dynamical quark masses at the two weakest couplings. Having added a fifth mass in each case and choosing some different fits of hadron propagators, some of the extrapolated masses have changed by two standard deviations. Thus, we urge some caution not to overreact to

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our dynamical results in the chiral limit.

Some details of the configuration generation and propagator calculation may be found in Ref. [1]. For the dynamical runs, the minimum values of m_π/m_ρ are 0.48 and 0.53 for $6/g^2 = 5.5$ and 5.6, respectively. For the stronger couplings, we have gone below 0.4.

We have found the chiral extrapolation for the quenched simulations to be a most vexing problem. Quenched chiral perturbation theory (QχPT) indicates that there are corrections to the nucleon and ρ masses that do not occur with dynamical quarks. The chiral expansion is commonly expressed in terms of the quark mass. For the quenched case, the terms $m_q^{1/2}$ and $m_q \log m_q$ are the new terms. (They are due to η' loops.) However, we note that due to flavor symmetry breaking for staggered quarks, the flavor singlet pion that appears in QχPT does not actually have a mass proportional to $m_q^{1/2}$. Thus, it is actually more appropriate to express the chiral expansion using a term proportional to a non-Goldstone pion mass. We define, for fixed λ_N,

$$m'_N \equiv (m_N + \lambda_N m_{\pi_2}) \frac{m_N^{\rm phys}}{m_N^{\rm phys} + \lambda_N m_\pi^{\rm phys}}, \qquad (1)$$

where phys stands for the physical values and the other quantities are values computed at a given quark mass and lattice spacing. A similar equation applies to the ρ. We then fit m'_N and m'_ρ using only terms that appear in ordinary χPT ($M + am_q + bm_q^2$ plus either $cm_q^{3/2}$ or $cm_q^2 \log m_q$) for various values of λ_N and λ_ρ obeying $0 \le \lambda_N \le \lambda_\rho \le 0.4$, which is the range expected from QχPT. Further details of the chiral and continuum extrapolations may be found in Ref. [2].

For the dynamical quark calculations, we tried six fits that only contain terms appropriate to ordinary χPT. The same four parameter forms we used above give good combined confidence level for the nucleon of 0.55 and 0.41. However, for the ρ the combined CL is quite poor. The poor fits occur for the strongest couplings. At $6/g^2 = 5.3$, in particular, there are two points that lie off a smooth curve. So far, we have picked fits independently at each run; however, we expect that the appropriate fitting range should be a smooth function of the quark mass. We may adjust our fitting ranges with this in mind.

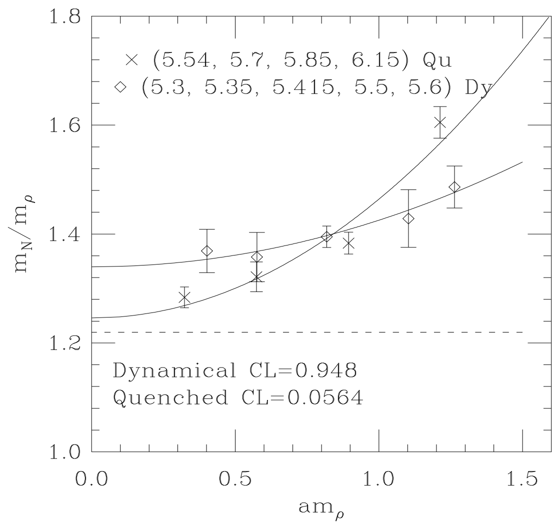

Figure 1. Continuum extrapolation for m_N/m_ρ with the physical quark mass.

The final extrapolation is in the lattice spacing. The leading error for Kogut-Suskind quarks is $O(a^2)$. For the quenched case, we must consider different values of λ_N and $lambda_\rho$, and the systematic error comes from this variation, as well as considering a possible quartic lattice spacing error, or a contamination from the π_2 that results in a linear error. When we combine all these systematic errors in quadrature, we find our final result $m_N/m_\rho = 1.254 \pm 0.018 \pm 0.028$.

For the dynamical quark results, $\lambda_N = \lambda_\rho = 0$, so the analysis is more straightforward. Note, however, that the lattice spacing changes with quark mass. It is therefore helpful to fix m_π/m_ρ (and hence the quark mass) to various values and then to do the continuum extrapolation separately for each choice of m_π/m_ρ. We compare the dynamical case with the quenched by letting $\lambda_N = \lambda_\rho = 0$ and using the same fitting function for N and ρ. (The terms specific to QχPT are only large for very small quark mass.) In Figs. 1 and 2. we compare quenched and dynamical results at $m_\pi/m_\rho = 0.1753$ and 0.5 with continuum fits that include an error quadratic in the lattice spacing.

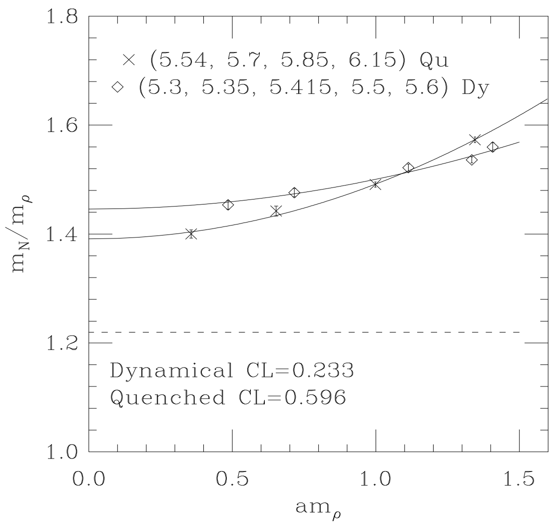

Figure 2. Continuum extrapolation for m_N/m_ρ with $m_\pi/m_\rho = 0.5$.

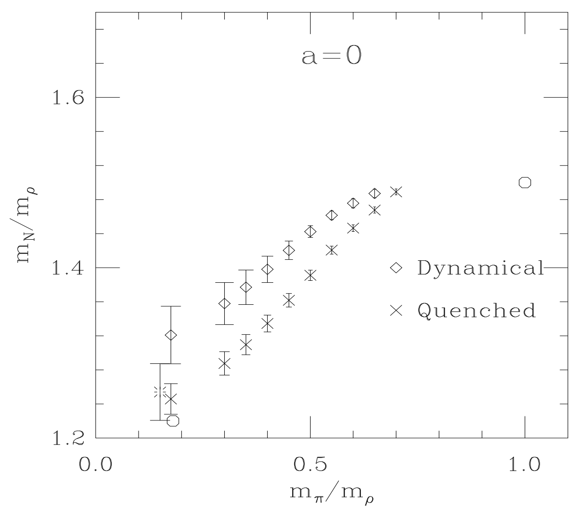

Figure 3. Edinburgh plot in the continuum limit for $N_f = 0$ and 2.

In Fig. 3, we compare the quenched and dynamical results in the continuum limit. (The burst, slightly displaced to the left, includes both systematic and statistical errors [2].) The dynamical results are above the quenched results for the entire range of m_π/m_ρ we consider. For $m_\pi/m_\rho = 0.55$ there is clearly a significant difference between the two (0.041 ± 0.007). This is a value where we don't have to extrapolate in quark mass. Also, if the quark mass were really this heavy, the ρ would not decay. We are seeing a real difference between the two flavor dynamical spectrum and the quenched approximation. Of course, we are especially interested in the chiral limit, because there we can compare with the real world. Recognizing that the quenched result for the physical value of m_π/m_ρ is closer to the experimental result, we are left to speculate why this is the case. First, we wish to again mention that for the dynamical case we have less confidence in the chiral limit because of the long extrapolation for the two weaker couplings. Second, as has always been the case with dynamical simulations, we take no special measures to deal with the potential for ρ decay. In lattice simulations, the threshold for decay is changed by finite volume effects. Perhaps the lattice suppression of the decay changes the ρ mass, or a complete analysis of the ρ and two π channels is required to correctly determine the ρ mass. Third, we only have two dynamical quarks in our calculation. Perhaps the strange quark plays a larger role than we might have expected.

Although much additional work remains, particularly regarding the chiral extrapolation, we are starting to obtain reliable results for lattice QCD, even with dynamical fermions, in the continuum limit. We see a very interesting difference between the $N_f = 2$ theory and the quenched approximation even in a region where only chiral interpolation is necessary.

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